Aerodynamic characteristics of rectangular flat plates with aspect ratios 5 : 1, 2 : 1, 1 : 1, 1 : 2 and 1 : 5 for use as windmill vane blades

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(C_l-α, C_d-α and C_m-α graphs added)

KD 551

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Contains page

1 Introduction

In report R443D (ref. 1) of the former Wind Energy Group of the University of Technology Eindhoven, about eighty airfoils are assembled which have been measured for low Reynolds numbers. A problem with this report is that it is no longer available and that most of the given airfoil graphs are too small for use in rotor blade calculations. Fortunately the original measuring points are given for almost all airfoils and using these points, new accurate graphs can be made.

Rectangular flat plates are normally not used for rotor blades because the minimum C_d/C_l ratio is too high. However, they can be used for vane blades very well. The combination of a vane blade and a vane arm is called a vane. Vanes can have two different functions. Main vanes keep the rotor and the head of a windmill in the wind and auxiliary vanes or side vanes push the rotor and the head out of the wind. Side vanes are used when the rotor axis has no eccentricity with respect to the tower axis. Main vane blades with many different shapes are used but for most shapes no aerodynamic characteristics are available and in this case the aerodynamic behaviour of the vane can only be found by try and error.

A main vane is normally used to keep the rotor in the wind but the combination of the main vane and an eccentric rotor or a main vane and a side vane can also be used as a safety system to turn the rotor out of the wind at high wind speeds. In report KD 485 (ref. 2) five different safety systems are described which turn the rotor out of the wind at high wind speeds.

A vane blade is normally positioned with the vane blade area vertically. The aspect ratio of a vane blade is the ratio in between the vane blade height h and the vane blade width w. So for a vane blade with an aspect ratio 1 : 2, the hight is half the width. If the plate is seen as an aerodynamic airfoil, the width w corresponds to the chord c and height h corresponds to the length l. Aerodynamic airfoils are measured in a wind tunnel with the length l horizontally.

The characteristics of a vane blade are normally given as the variation of the lift coefficient C_l and the drag coefficient C_d as a function of the angle of attack α. These parameters are defined in chapter 3 of report KD 35 (ref. 3). However, for a vane blade, one is normally only interested in the component of lift L and drag D perpendicular to the vane blade. This component is called the normal force N and the coefficient is called the normal coefficient C_n. The C_n-α curves for rectangular plates with aspect ratios 5 : 1, 1 : 1 (square plate) and 1 : 5 are given in figure 13 of report R999D (ref. 4). The C_n-α curves for rectangular plates with aspect ratios 2 : 1 and 1 : 2 are given in figure 31 of report R999D. I have a copy of this report but as this report is no longer available, it seems useful to put the aerodynamic characteristics of rectangular plates with these five aspect ratios together in this report KD 551.

The aerodynamic characteristics of rectangular flat plates with aspect ratios 5 : 1, 1 : 1 and 1 : 5 are also given in report R443D (ref. 1). I have a copy of this report but also this report is no longer available. The measurements are originally performed by Flagsbart. The original measurements were published in: Messungen an ebenen und gewölbten platten (in German, ref. 5). The characteristics are given as C_l/C_d curves and C_l/C_m curves in R443D but the original measuring points are also given in tables as a function of α, so it is easy to derive the wanted curves. The characteristics for a square plate are almost identical to the characteristics of a circular plate. The characteristics for square plates were measured for four different Reynolds values $2 * 10^5$, $4 * 10^5$, $6 * 10^5$ and $8 * 10^5$ but no different characteristics are given. This means that the characteristics for square plates are almost independent of the Reynolds number. The characteristics for rectangular plates with aspect ratios 5 : 1 and 1 : 5 were measured only for the Reynolds value $4 * 10^5$.

The characteristic for rectangular plates with aspect ratios 2 : 1 and 1 : 2 were measured by a student in the wind tunnel of the department of Physics of the University of Technology Eindhoven. The results are presented in report R487S (ref. 6).

However, I don't have a copy of this report and so the original measuring points aren't available. So the C_n-α curves of figure 31 of report R999D were copied as accurately as possible. The original C_l-α and C_d-α characteristics from which the C_n-α curves are derived are also not available. It isn't known for which Reynolds values the measurements are valid but I think it will be around $2 * 10^5$.

2 The square flat plate

The table with measuring points for the square flat plate is given at page 3-4 of R443D (ref. 1). This table is copied as table 1. Apart from the lift coefficient C_l and the drag coefficient C_d, the moment coefficient C_m is also given. The aerodynamic moment M and the moment coefficient C_m are normally taken around the quart chord point but for the square flat plate they are taken around the airfoil nose. The lift L and the drag D also have the airfoil nose as point of application. The left hand direction of the aerodynamic moment is positive if the wind is coming from the left side and if the angle of attack α is positive (see figure 1).

The lift force L and the drag force D can be dissolved into a component perpendicular to the plate and in a component in line with the plate. The sum of the components of L and D perpendicular to the plate is the normal force N. The normal coefficient C_n is given by:

$$C_n = C_l * \cos \alpha + C_d * \sin \alpha \qquad (-) \qquad (1)$$

The difference in between the drag component $C_d * \cos \alpha$ and the lift component $C_l * \sin \alpha$ in the direction of the plate is small and can be neglected, also because this force gives no moment around the turning axis of the vane arm. The calculated C_n values using formula 1 are also given in table 1. For a vane blade, it isn't easy to calculate the vane moment around the turning axis of the vane arm, using the normal force N and the aerodynamic moment M. Therefore, the normal force N is assumed to have a point of application at a certain distance i from the nose (see figure 1). The distance i is chosen such that the moment produced by the normal force is equal to the aerodynamic moment M.

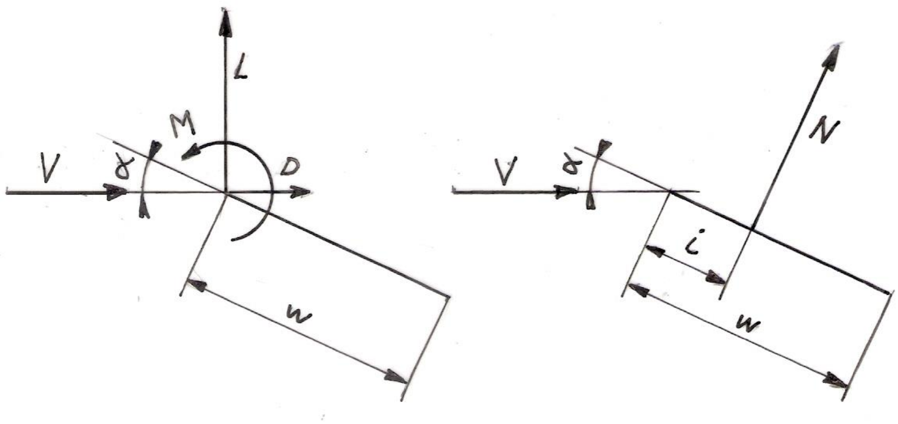

fig. 1 Replacement of the L, D and M by N at a distance i from the nose

It can be derived that:

$$i / w = C_m / C_n \qquad (2)$$

The calculated values of i / w are also given in table 1.

α (°)	C_l (-)	C_d (-)	C_m (-)	C_n (-)	i/w (-)
0	0	0.0232	0	0	-
5.0	0.161	0.0363	0.035	0.164	0.213
9.9	0.361	0.0842	0.098	0.370	0.265
14.9	0.591	0.176	0.193	0.616	0.313
19.9	0.831	0.313	0.299	0.885	0.338
24.6	1.015	0.479	0.402	1.122	0.358
34.7	1.300	0.904	0.606	1.583	0.383
37.7	1.330	1.026	0.668	1.680	0.398
39.7	1.327	1.100	0.708	1.724	0.411
40.7	1.323	1.101	0.724	1.721	0.421
37.9	0.887	0.703	0.478	1.132	0.422
39.9	0.840	0.709	0.463	1.099	0.421
40.9	0.832	0.722	0.467	1.102	0.424
41.9	0.821	0.737	0.480	1.103	0.435
46.9	0.751	0.799	0.472	1.097	0.430
54.9	0.655	0.925	0.493	1.133	0.435
64.4	0.484	1.020	0.505	1.129	0.447
75.0	0.302	1.085	0.528	1.126	0.469
90.0	0	1.150	0.566	1.150	0.492

table 1 C_l, C_d, C_m, C_n and i / w for a square plate as a function of α

The measured C_l-α, the C_d-α and the C_m-α curves are given in figure 2, 3 and 4.

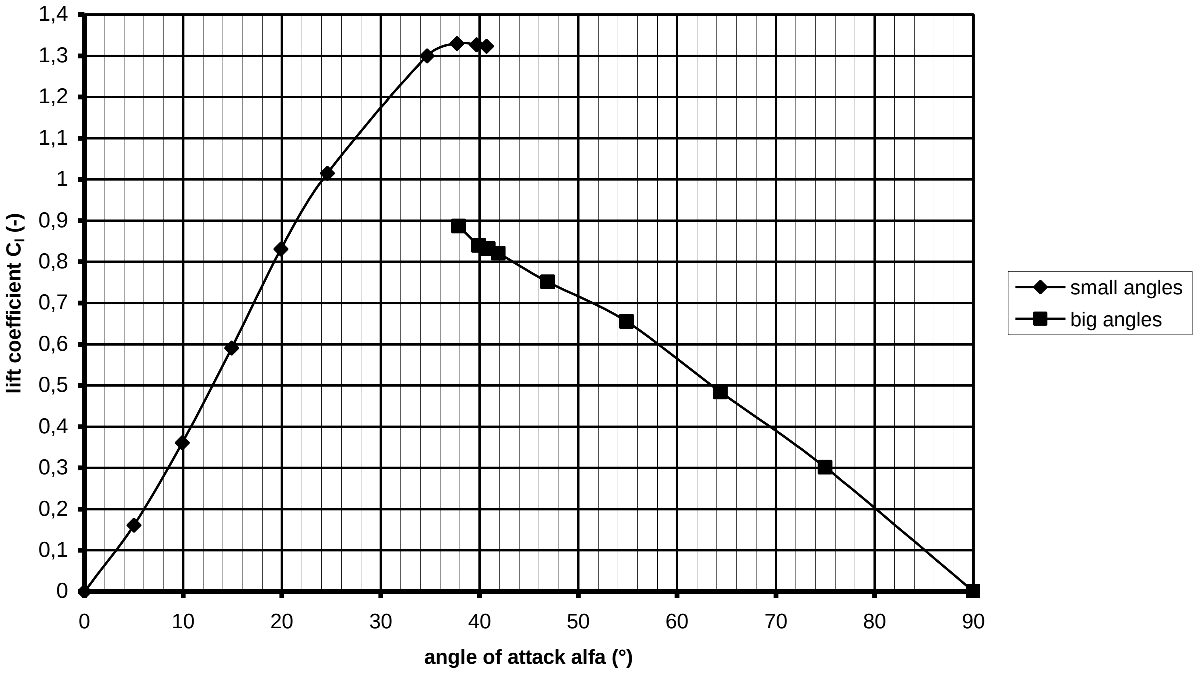

fig. 2 Measured values of C_l as a function of α for a square flat plate

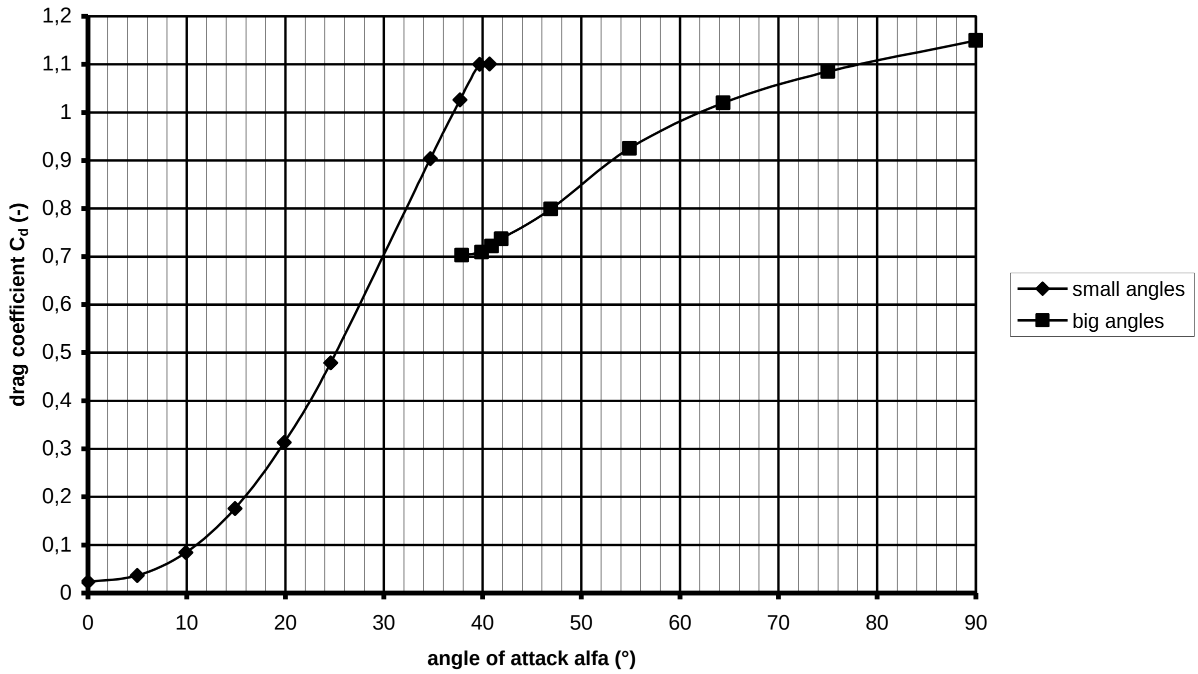

fig. 3 Measured values of C_d as a function of α for a square flat plate

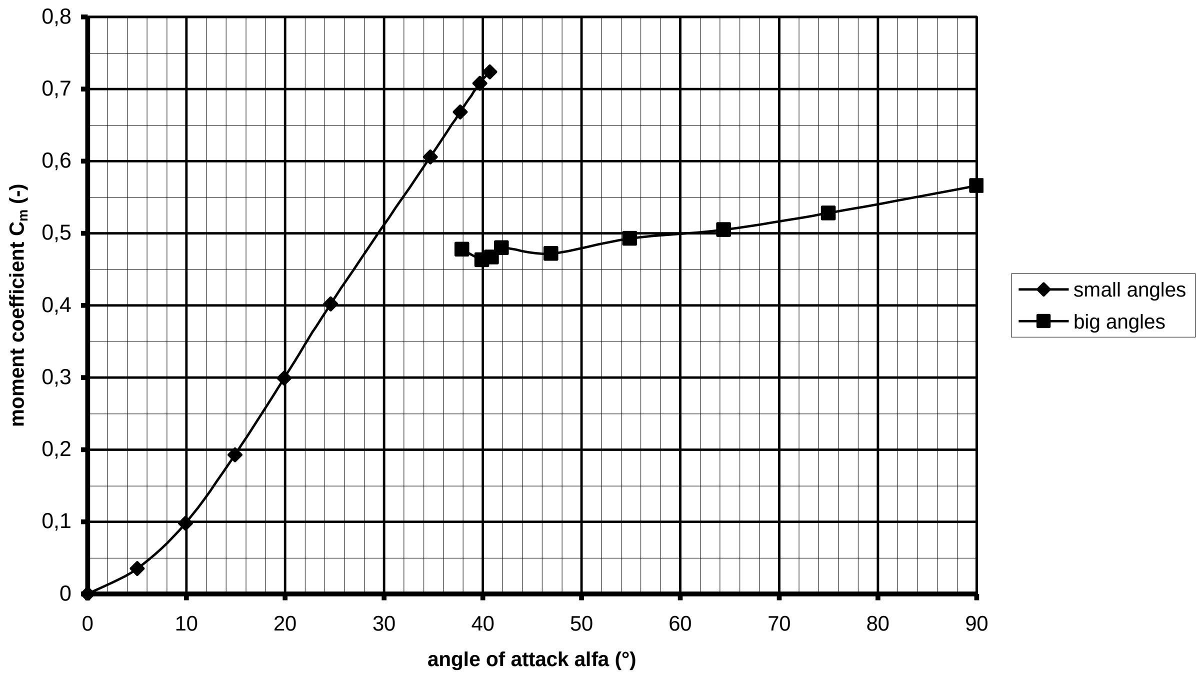

fig. 4 Measured values of C_m as a function of α for a square flat plate

The calculated values of C_n as a function of α are given in the C_n-α curve of figure 5.

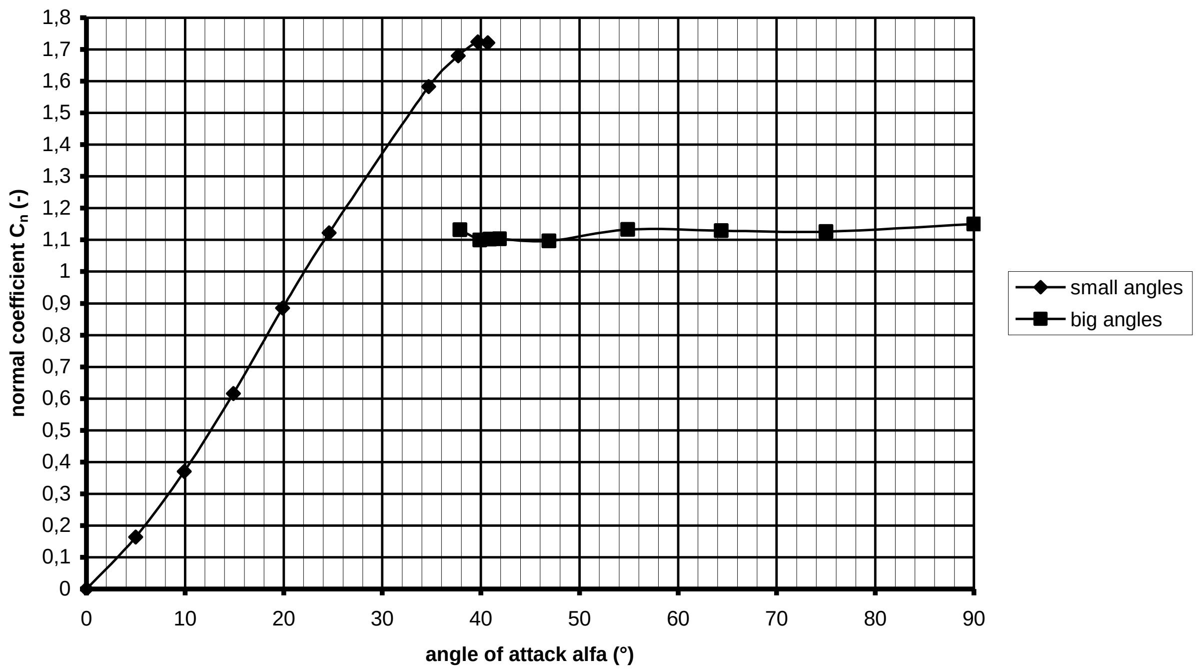

fig. 5 Calculated values of C_n as a function of α for a square flat plate

The calculated values of i / w as a function of α are given in the i/w-α curve of figure 6.

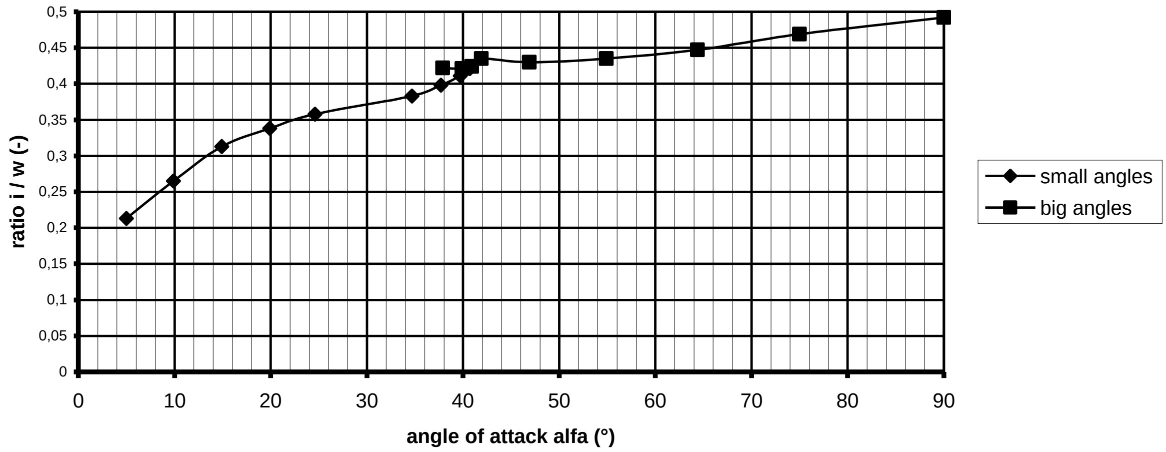

fig. 6 Calculated values of i / w as a function of α for a square flat plate

In figure 5 it can be seen that C_n is increasing about linear with α for $\alpha < 40°$ and that C_n is about constant for $\alpha > 40°$. There is hysteresis for the region $37.9° < \alpha < 40.7°$. In this region, C_n suddenly drops at $\alpha = 40.7°$ if the left curve is followed by increasing α. C_n suddenly increases at $\alpha = 37.9°$ if the right curve is followed by decreasing α.

In figure 6 it can be seen that the ratio i / w is slowly increasing at increasing values of α. The vane blade will normally be used for angles of α in between about 10° and 35° and the average value of i / w for this region is about 0.35. The length of the vane arm R_v from the turning axis up to the nose of the vane blade will be about three times the vane width w. So the distance (R_v + i) in between the turning axis and N is about 3.35 w. For this length it is allowed to calculate the vane moment M_v around the turning axis of the vane for a constant value i = 0.35 w because the variation of i has only a little influence on the total value of (R_v + i).

For the vane moment M_v it is valid that:

$M_v = N * (R_v + i)$ (Nm) (3)

The normal force N is given by:

$N = C_n * ½ \rho V^2 * w * h$ (N) (4)

In this formula ρ is the density of air (about 1.2 kg/m^3 at a temperature of 20 °C at sea level) and V is the wind speed (in m/s) felt by the vane blade.

3 The rectangular flat plate for aspect ratios 2 : 1 and 1 : 2

The C_n-α curves of an aspect ratio of 2 : 1 and of 1 : 2 are given in one graph as figure 31 of report R999D (ref. 4). As no measuring point are available, these curves were copied as accurate as possible in figure 7.

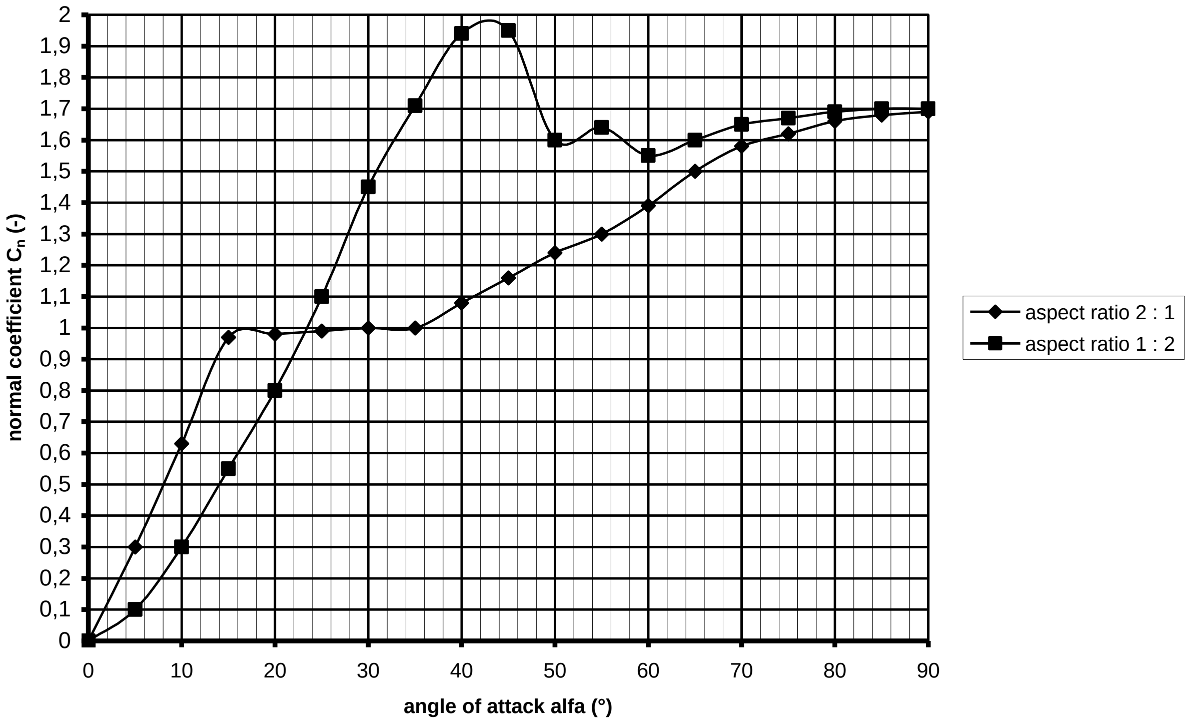

fig. 7 C_n-α curves for rectangular flat plates with aspect ratios 2 : 1 and 1 : 2

In figure 7 it can be seen that there is a large difference in between the C_n-α curves for aspect ratios 2 : 1 and 1 : 2. The C_n-α curve for an aspect ratio 2 : 1 has only a rather short linear part up to about α = 15°, so this aspect ratio is not advised if the vane is used as main vane. But for a side vane this aspect ratio is much better because there is no big peak in the characteristic which will cause turning out of the wind suddenly if the peak is reached. The C_n-α curve for an aspect ratio of 1 : 2 has a large almost linear part up to about α = 42° and the maximum C_n value is very high (almost 2). So a vane blade with an aspect ratio of 1 : 2 is well suited for a main vane.

I have strong indications that the measurements performed in the wind tunnel of Eindhoven are unreliable so one should be careful to use the graphs of figure 7 (see explanation in chapter 5).

4 The rectangular flat plate for aspect ratios 5 : 1 and 1 : 5

The table with measuring points for the rectangular flat plates with aspect ratios 5 : 1 and 1 : 5 are given at page 3-3 of R443D (ref. 1). These tables are copied as table 2. The C_n values and the ratio i / w are calculated in the same way as done for the square flat plate and are also added to table 2.

aspect ratio 5 : 1						aspect ratio 1 : 5					
α (°)	C_l (-)	C_d (-)	C_m (-)	C_n (-)	i/w (-)	α (°)	C_l (-)	C_d (-)	C_m (-)	C_n (-)	i/w (-)
0	0	0.0218	0	0	-	0	0	0.0066	0	0	-
4.9	0.377	0.0450	0.092	0.379	0.243	5.0	0.063	0.0112	0.015	0.064	0.234
9.7	0.719	0.135	0.258	0.731	0.353	10.0	0.147	0.0262	0.038	0.149	0.255
14.7	0.774	0.219	0.313	0.804	0.389	14.9	0.300	0.0860	0.105	0.312	0.337
17.6	0.807	0.268	0.330	0.850	0.388	19.8	0.435	0.168	0.154	0.466	0.330
19.7	0.817	0.305	0.340	0.872	0.390	23.8	0.582	0.290	0.217	0.650	0.334
23.8	0.805	0.369	0.361	0.885	0.408	29.7	0.725	0.438	0.282	0.847	0.333
26.7	0.832	0.424	0.378	0.934	0.405	34.7	0.824	0.608	0.335	1.029	0.326
29.7	0.880	0.497	0.415	1.011	0.410	39.7	0.870	0.755	0.374	1.152	0.325
30.7	0.871	0.511	0.412	1.010	0.408	42.2	0.880	0.827	0.402	1.207	0.333
31.7	0.872	0.534	0.405	1.023	0.396	44.7	0.880	0.898	0.425	1.257	0.338
33.2	0.861	0.563	0.418	1.029	0.406	49.7	0.840	1.030	0.476	1.329	0.358
34.7	0.835	0.578	0.418	1.016	0.411	54.7	0.780	1.080	0.506	1.332	0.380
37.7	0.802	0.608	0.421	1.006	0.418	55.7	0.750	1.090	0.513	1.323	0.388
40.7	0.754	0.648	0.409	0.994	0.411	57.2	0.702	1.080	0.519	1.288	0.403
44.7	0.720	0.707	0.426	1.009	0.422	59.8	0.643	1.070	0.526	1.248	0.421
49.8	0.679	0.791	0.452	1.042	0.434	64.8	0.530	1.070	0.543	1.194	0.455
54.8	0.629	0.866	0.472	1.070	0.441	69.8	0.432	1.100	0.549	1.182	0.464
59.8	0.544	0.952	0.468	1.096	0.427	74.9	0.341	1.130	0.557	1.180	0.472
69.8	0.415	1.073	0.528	1.150	0.459	79.9	0.247	1.159	0.564	1.184	0.476
79.9	0.233	1.163	0.558	1.186	0.470	84.9	0.148	1.180	0.577	1.188	0.486
85.0	0.128	1.190	0.565	1.197	0.472	90.0	0	1.200	0.593	1.200	0.494
90.0	0	1.200	0.578	1.200	0.482						

table 2 C_l, C_d, C_m, C_n and i / w for rectangular flat plates as a function of α for aspect ratios 5 : 1 and 1 : 5

The measured C_l-α, the C_d-α and the C_m-α curves are given in figure 8, 9 and 10.

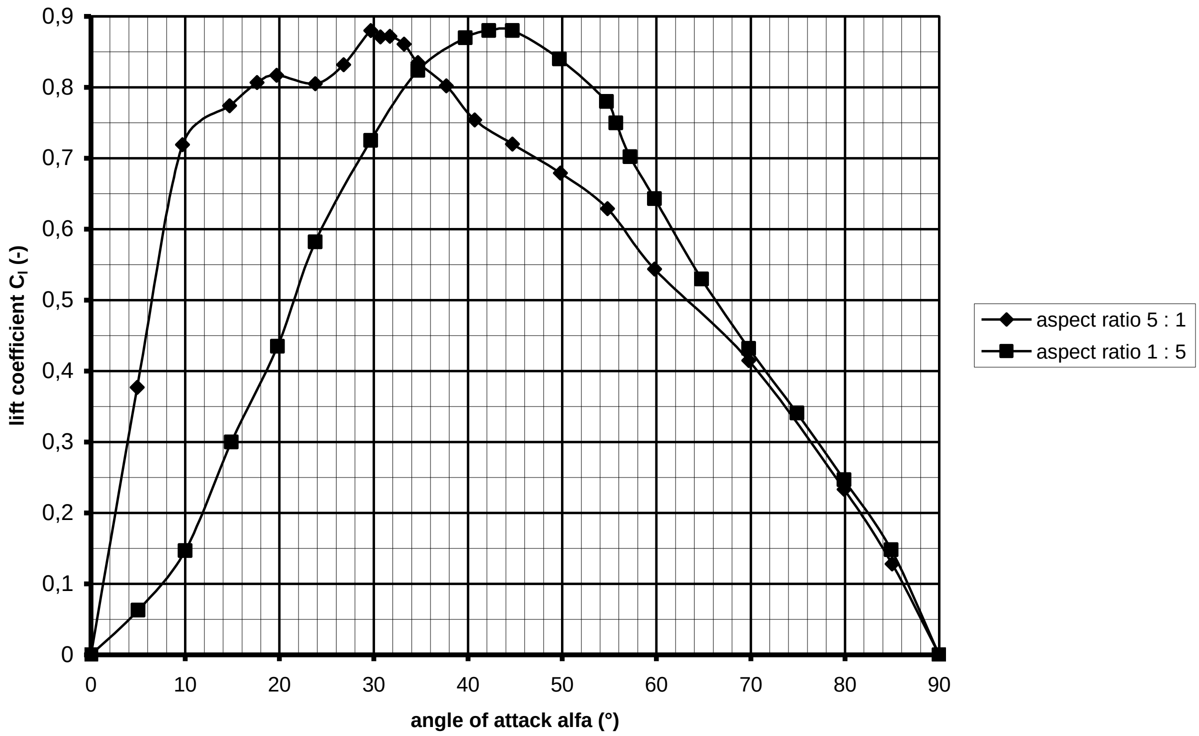

fig. 8 Measured values of C_l as a function of α for rectangular flat plates with ratios 5 : 1 and 1 : 5

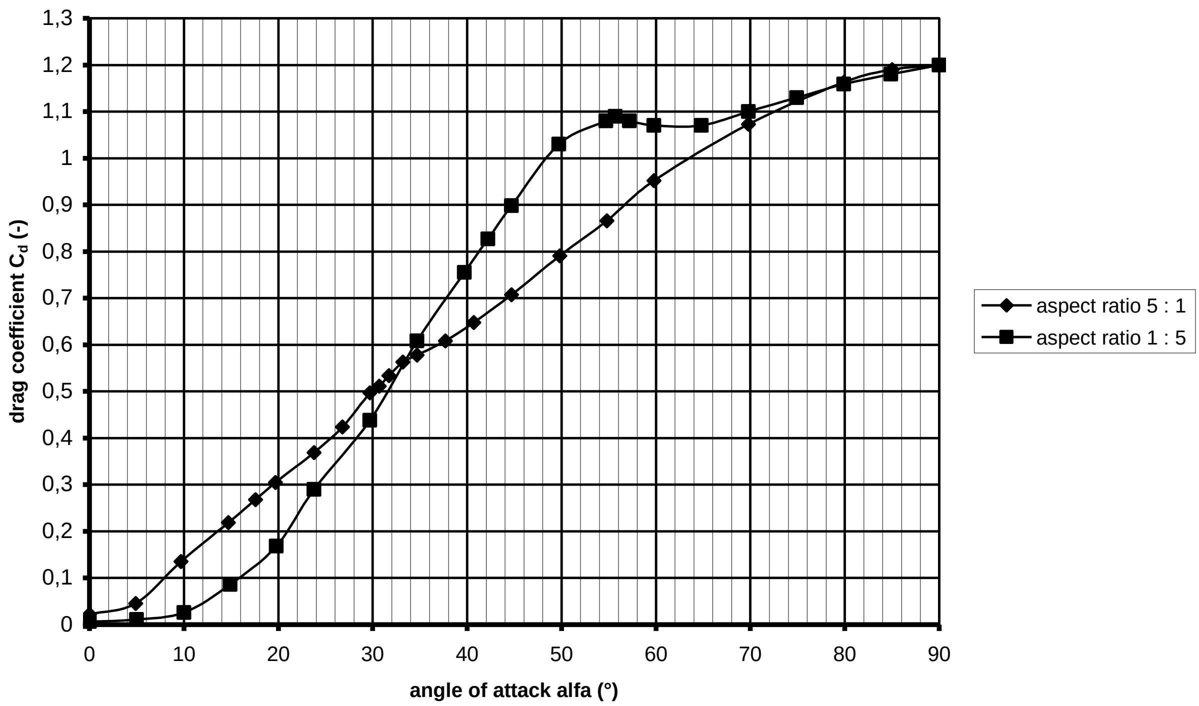

fig. 9 Measured values of C_d as a function of α for rectangular flat plates with ratios 5 : 1 and 1 : 5

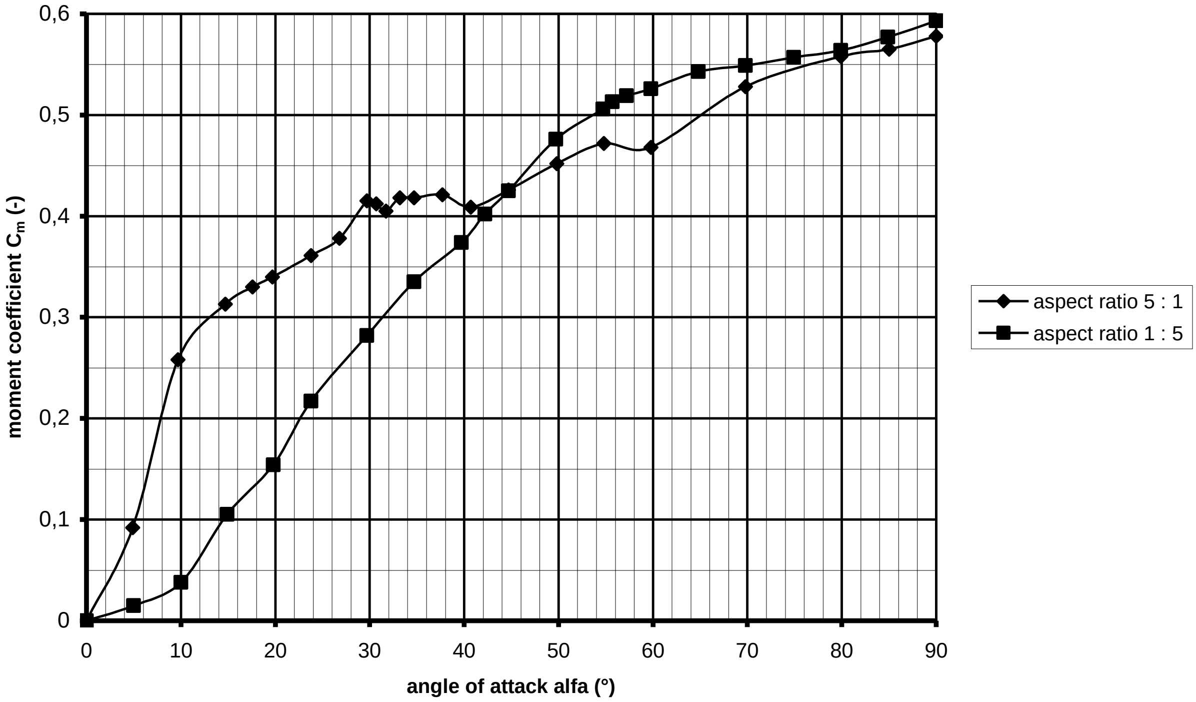

fig. 10 Measured values of C_m as a function of α for rectangular flat plates with ratios 5 : 1 and 1 : 5

The calculated values of C_n as a function of α are given in the C_n-α curves of figure 11.

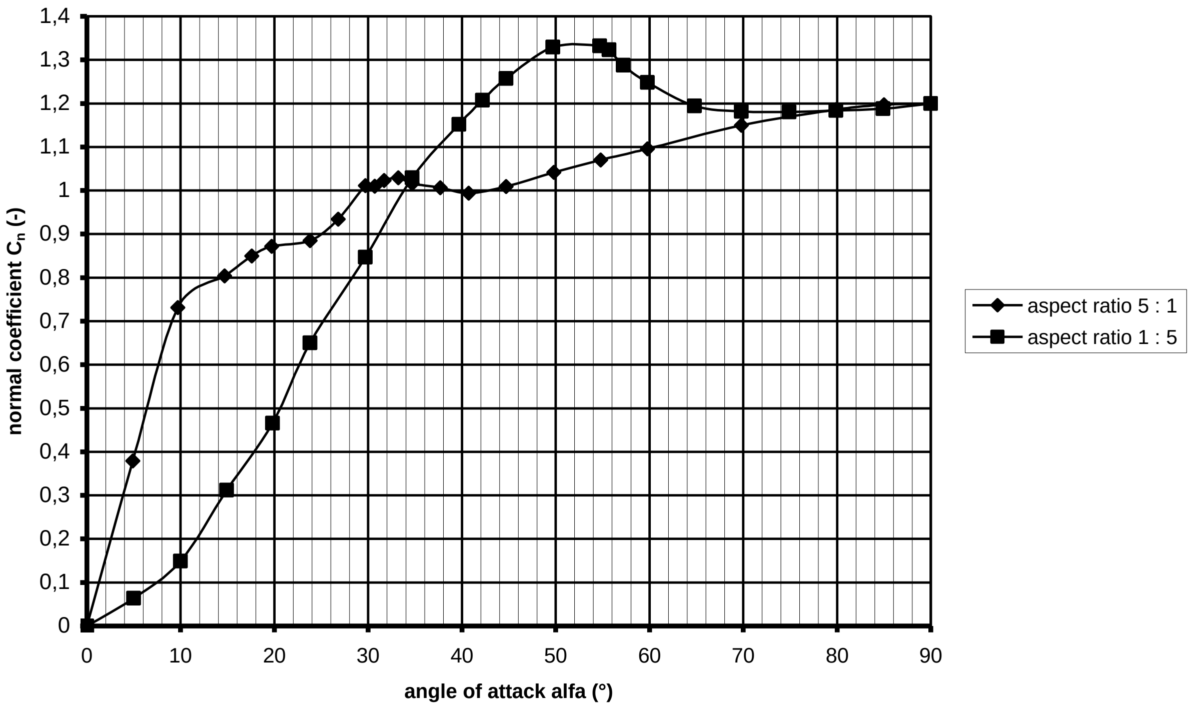

fig. 11 Calculated values of C_n as a function of α for rectangular flat plates with ratios 5 : 1 and 1 : 5

The calculated values of i / w as a function of α are given in the i/w-α curves of figure 12.

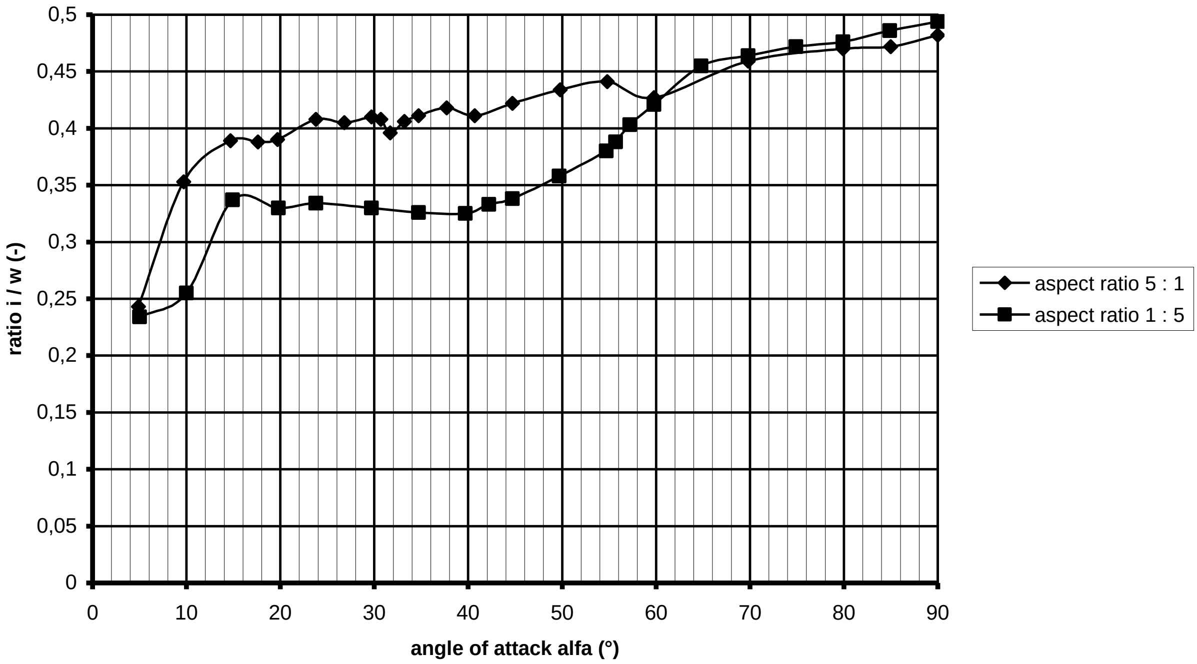

fig. 12 Calculated values of i / w as a function of α for rectangular flat plates with ratios 5 : 1 and 1 : 5

Also for aspect ratios 5 : 1 and 1 : 5 there is a large difference in between the characteristics. But the maximum C_n value for 1 : 5 is much lower than for 1 : 2 and 1 : 1 and the aspect ratios 1 : 2 or 1 : 1 are therefore preferred as main vane.

5 Comparing of the measurements

The measurements for the square flat plate and for the rectangular flat plates with aspect ratios 5 : 1 and 1 : 5 have been measured by the same man (Flagsbart) and in the same wind tunnel in Göttingen. This is a very famous wind tunnel and it can be expected that one has taken care to prevent tunnel blockage by the object which was measured. So I believe that these measurements are reliable.

If the values of C_n are compared for α = 90° it can be seen that for the square plate, C_n = 1.15 and that for the rectangular flat plate with aspect ratios 5 : 1 and 1 : 5, that C_n = 1.2, so a little higher. However, for the rectangular flat plate with aspect ratio 2 : 1, C_n = 1.69 and for the aspect ratio 1 : 2, C_n = 1.7. These values should be the same but the most strange fact is that the values are much larger than for a square plate. I don't believe that this is correct so a did a simple test to check my doubts.

The C_n value for α = 90° is the same as the drag coefficient for α = 90° as no lift will be produced for this angle. So one can compare the drag coefficients for α = 90°. To do this I made an aerodynamic balance. The balance consist of a beam which can turn around a small vertical tower which has the axis in the centre of the beam. On one side a rectangular flat plate is connected with a width of 100 mm and a height of 200 mm, so with an aspect ratio of 2 and an area of 20000 mm^2. On the other side a square plate is connected with a height and width of 141.4 mm, so also with an area of 20000 mm^2. The distance in between the hart of both plates and the centre of the beam is 242 mm for both plates.

The tower is hold in one hand and I was driving on a bicycle against the wind. What happened was that the balance staid perpendicular to the wind. So this means that the drag coefficient of a rectangular flat plate with an aspect ratio 2 : 1 is just the same as that of a square flat plate.

I even noticed that the square flat plate wins the game if a small yaw angle is created. This can be understood because the C_n value for a square plate is almost constant for large angles α but for a rectangular plate with aspect ratio 2 : 1, C_n decreases if the angle decreases from 90° up to 60°.

This test demonstrates that the values measured in the wind tunnel of the department of Physics the University Technology of Eindhoven are much too high. I was working at the Wind Energy Group of this department at that time and there were other indications that the measured values were too high. It may have the following reason. The wind tunnel has a square measuring section of 0.5 m * 0.5 m which is rather small. So if one wants to measure an object in a closed measure section, these object must be very small to prevent tunnel blockage. The measure section is therefore removed, so one is measuring in an open flow. This allows expansion of the wake around the object. However, this expansion is not as much as for a full open wind tunnel or for a big wind tunnel with a small object, because the air is sucked in the tunnel opening at the back side of the object.

So this test demonstrates that the C_d and so the C_n value for α = 90° is a factor 1.69 / 1.15 = 1.47 too high! The tunnel blockage is maximal for the vane blade perpendicular to the air flow. It can be expected that the blockage is lesser for small angles α. However, this test demonstrates that the measurements executed for the aspect ratios 2 : 1 and 1 : 2 are very unreliable, especially for large angles α!

6 References

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