Calculations executed for the rotor of the VIRYA-3.1 windmill (λ_d = 6.5, Gö 624 airfoil), with 2-bladed rotor made of Roofmate and glass fibre

ing. A. Kragten

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KD 511

The rotor should not be used if the windmill has no safety system which turns the rotor fluently out of the wind at high wind speeds!

Engineering office Kragten Design
Populierenlaan 51
5492 SG Sint-Oedenrode
The Netherlands
telephone: +31 413 475770
e-mail: info@kdwindturbines.nl
website: www.kdwindturbines.nl

Contains page

1 Introduction

Already in 1977 I have designed and built a very light 2-bladed windmill rotor with a diameter D = 3.2 m and a design tip speed ratio λ_d = 8 which was manufactured from the isolation material Roofmate covered with glass fibre and epoxy. A photo of this rotor is given in appendix 1. This rotor was damaged in a strong storm because the windmill for which it was used, was not provided with a safety system which limits the thrust and because the rotor was not strong enough. However, the manufacturing procedure was rather simple and a large rotor can be manufactured with the used procedure at a rather low price. So it is investigated how a rather large rotor looks like if it is manufactured from Roofmate material.

Roofmate is standard supplied in sheets with a length of 1250 mm and a width of 600 mm. It can be supplied in a large range of thicknesses. There is some tolerance on the length and width, so a sheet is shortened accurately to a length of 1240 mm.

A rotor is made of four 620 mm long blade sections and one 620 mm long central section resulting in a rotor diameter of 3.1 m. The windmill is called the VIRYA-3.1.

The VIRYA-3.1 is an alternative of the VIRYA-3 rotor with two wooden blades or the VIRYA-3B2 rotor with two steel blades. The generator, the head and the tower will be identical to those of the VIRYA-3. Measurements of the generator are given in report KD 78 (ref. 1).

2 Description of the rotor of the VIRYA-3.1 windmill

A blade is made of two 0.62 m long segments which are glued together by epoxy. Two rotor blades are glued to a central part which is also made from Roofmate and which also has a length of 0.62 m. So the rotor radius R = 1.55 m. The whole rotor is covered with some layers of glass fibre imbedded in epoxy to make the rotor strong and stiff enough.

A blade segment is made of a 0.62 m long part of Roofmate by connecting a jig with the required airfoil to both sides of the sheet. The blade segment is cut from the sheet by moving a hot wire simultaneously along both jigs. This procedure allows a small blade twist in between both sides of the blade segment.

The 2-bladed rotor of the VIRYA-3.1 windmill has a design tip speed ratio λ_d = 6.5. This is lower than the λ_d = 8 of the 3.2 m rotor made in 1977 and the chords are therefore relatively larger resulting in a stronger rotor. The most important advantages of a 2-bladed rotor above a 3-bladed rotor are that the connection of the blades to the hub is simple and that balancing of the rotor is rather simple. The rotor can be transported completely mounted.

The rotor has blades with a linearised chord and blade angle and is provided with a rather thick Gö 624 airfoil to make the blades strong enough. The geometry and characteristics of the Gö 624 airfoil are given in report KD 463 (ref. 2). The maximum thickness is 16 % of the chord. The lower side of the Gö 624 airfoil is flat over 70 % of the chord which simplifies manufacture. The central part has a rectangular cross section in the middle but long transistion parts to realise a fluent transistion to the airfoil geometry.

The chord is 160 mm at the blade tip and 240 mm at the blade root. The increase of the chord and so also of the blade thickness, results in a strong increase of the blade strength.

An aluminium disk with a diameter of about 200 mm is glued to both sides of the central part to guide the forces acting on the rotor to the hub. The rotor is bolted to a stainless steel hub by four bolts M8. The hub is connected to the tapered shaft end of the generator by one bolt M10. A sketch of the rotor is given in appendix 2.

The blades will be rather flexible and therefore vibrations which are caused by the gyroscopic moment, by streaming under a certain yaw angle δ and by a non-uniform distribution of the wind speed over the rotor plane, are flattened. The rotor is balanced by adding some extra glass and epoxy to the lightest blade tip or by gluing small cylinders of lead in holes which are drilled in the lightest blade tip.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 511) has its own formula numbering. Substitution of $\lambda_d = 6.5$ and R = 1.55 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 4.1935 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 2 in formula (5.4) of KD 35 gives:

$C_l = 12.566\ r\ (1 - \cos\phi) / c$ (-) (4)

Substitution of V = 5 m/s in formula (5.5) of KD 35 gives:

$R_{e\,r} = 3.335 * 10^5 * c * \sqrt{(\lambda_{r\,d}^2 + 4/9)}$ (-) (5)

The blade is calculated for nine stations A till I which have a distance of 0.155 m (so a quart of a blade section) of one to another. First the theoretical values are determined for C_l, α and β. Next β is linearised such that the twist is constant in between the ends of a blade section and that the linearised values correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The Reynolds values for the stations are calculated for a wind speed of 5 m/s because for most working hours, the windmill will be used at rather low wind speeds.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\ th}$ (-)	$C_{l\ lin}$ (-)	$R_{e\ r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ Gö 624	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\ lin}$ (-)
A	1.55	6.5	5.8	0.16	0.63	0.65	3.49	4.2	-0.5	-0.4	6.3	6.2	0.019
B	1.395	5.85	6.5	0.17	0.66	0.66	3.34	4.2	-0.2	-0.2	6.7	6.7	0.019
C	1.24	5.2	7.3	0.18	0.69	0.70	3.15	2.2	0.0	0.1	7.3	7.2	0.031
D	1.085	4.55	8.3	0.19	0.75	0.74	2.91	2.2	0.7	0.6	7.6	7.7	0.030
E	0.93	3.9	9.6	0.20	0.82	0.81	2.64	2.2	1.5	1.4	8.1	8.2	0.029
F	0.775	3.25	11.4	0.21	0.92	0.92	2.32	2.2	2.6	2.6	8.8	8.8	0.027
G	0.62	2.6	14.0	0.22	1.06	1.06	1.97	2.2	4.6	4.6	9.4	9.4	0.027
H	0.465	1.95	18.1	0.23	1.26	1.26	1.58	1.1	8.1	8.1	10.0	10.0	0.046
I	0.31	1.3	25.0	0.24	1.53	-	1.17	1.1	-	14.4	-	10.6	-

table 1 Calculation of the blade geometry of the VIRYA-3.1 rotor

No value for α_{th} and therefore for β_{th} is found for station I because the required C_l value can not be generated. β is linearised in between station A and E and in between station E and I. The twist per station is 0.5° in between stations A up to E and 0.6° in between stations E up to I. For these values, the linearised angles are lying close to the theoretical angles. A sketch of the rotor is given in appendix 2.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the outer seven stations of the blade is about 0.026. Figure 4.6 of KD 35 (for B = 2) and λ_{opt} = 6.5 and C_d/C_l = 0.027 gives $C_{p\ th}$ = 0.46. The blade is stalling in between station H and I. Therefore not the whole air foiled blade length k = 1.24 m, but only the part up to 0.14 m outside station I is taken into account for the calculation of the C_p. This gives k = k' = 1.1 m.
Substitution of $C_{p\ th}$ = 0.46, R = 1.55 m and blade length k = k' = 1.1 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.42. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.42 / 6.5 = 0.0646.
Substitution of $\lambda_{opt} = \lambda_d$ = 6.5 in formula 6.4 of KD 35 gives λ_{unl} = 10.4.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

Formula 6 is only valid for a blade with a constant chord and a constant blade angle but it gives a good approximation for a tapered blade if the values at half the blade length are used. Section E is lying at halve the blade length. The chord at section D is 0.2 m and the blade angle β = 8.2°. If the rotor is not rotating, the angle of attack α = 90° - β. So the average angle of attack is 90° - 8.2° = 81.8°.

The C_l-α curves for the Gö 624 airfoil are not given for large angles α. However, the airfoil is completely stalling during starting and it is expected that the C_l-α curve for large angles α will be about the same as the C_l-α curve for the Gö 623 airfoil. The C_l-α curve for the Gö 623 airfoil for large angles α is given in figure 5.10 of report KD 35 (ref. 4). For α = 81.8° it can be read in this figure that C_l = 0.29. During starting the whole blade length is stalling so the real air foiled blade length k = 1.24 is used for the calculation of $C_{q\ start}$.

Substitution of B = 2, R = 1.55 m, k = 1.24 m, C_l = 0.29 en c = 0.2 m in formula 6 gives that $C_{q\ start}$ = 0.0086. The real starting torque coefficient will be somewhat lower than the calculated value because we have used the average chord and the average blade angle. It is assumed that $C_{q\ start}$ = 0.008. For the ratio between the starting torque and the optimum torque we find that it is 0.008 / 0.0646 = 0.124. This is acceptable for a rotor met a design tip speed ratio of 6.5. The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \qquad (m/s) \qquad (7)$$

The sticking torque Q_s of the VIRYA-3 generator with 5RN90L04V housing has been measured and it was found that Q_s = 0.4 Nm. Substitution of Q_s = 0.4 Nm, $C_{q\ start}$ = 0.008, ρ = 1.2 kg/m^3 en R = 1.55 m in formula 7 gives that V_{start} = 2.7 m/s. This is acceptable for a 2-bladed rotor with a design tip speed ratio of 6.5.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 4). With this method, it can be determined that the C_q-λ curve is about straight and horizontal for low values of λ if a Gö 623 or a Gö 624 airfoil is used.

A scale model of a three bladed rotor with constant chord and blade angle and with a design tip speed ratio λ_d = 6 has been measured in the open wind tunnel of the University of Technology Delft already on 20-11-1980. It has been found that the maximum C_p was more than 0.4 and that the C_q-λ curve for low values of λ was not horizontal but somewhat rising. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-3.1 rotor are given in figure 1 and 2. The low C_q and C_p values at low values of λ are caused by stalling of the airfoil.

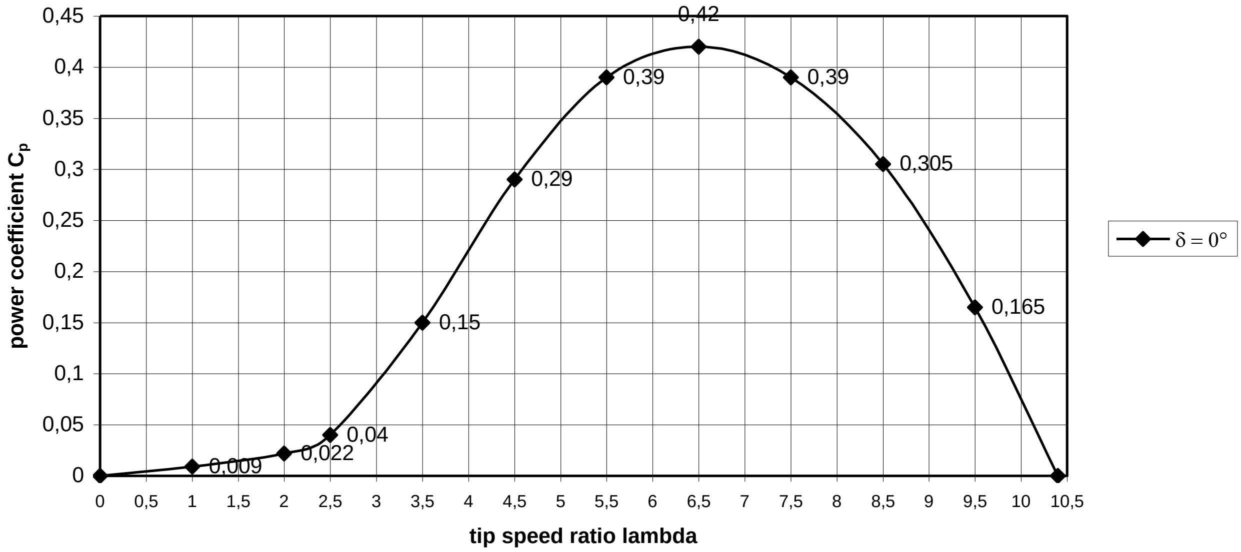

fig. 1 Estimated C_p-λ curve for the VIRYA-3.1 rotor for the wind direction perpendicular to the rotor (δ = 0°)

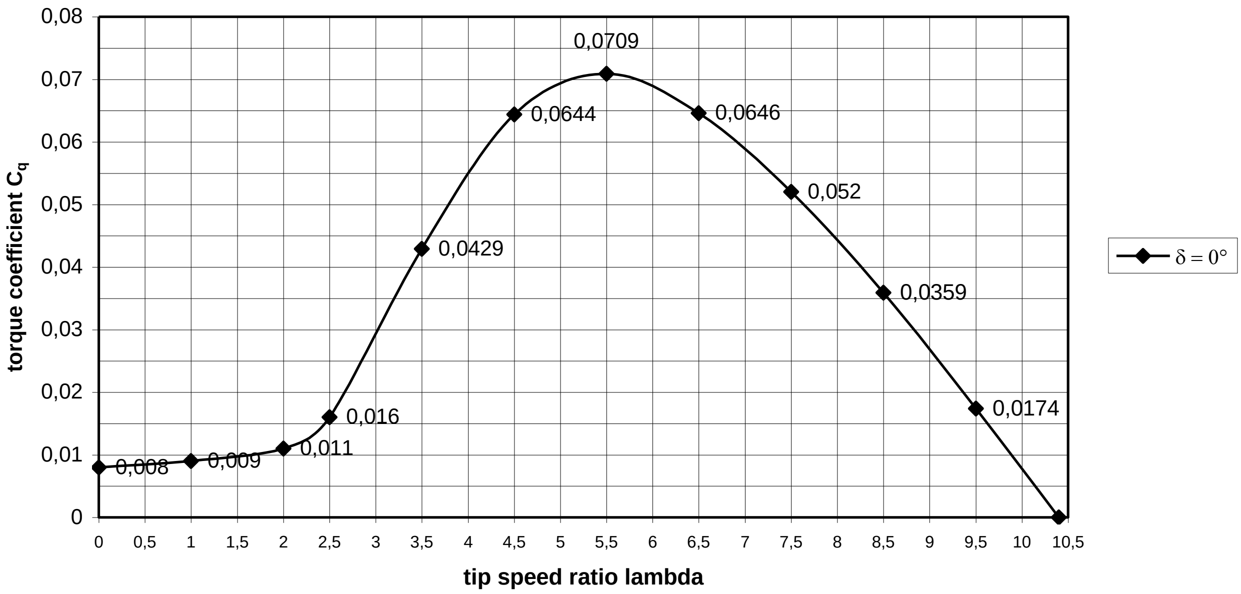

fig. 2 Estimated C_q-λ curve for the VIRYA-3.1 rotor for the wind direction perpendicular to the rotor (δ = 0°)

5 Determination of the P-n curves and the P_{el}-V curve for 26 V star

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and the δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The VIRYA-3.1 will be executed with the hinged side vane safety system which is described in report KD 213 (ref. 2) for the VIRYA-4.2 windmill. Both windmills have a 9 mm plywood vane blade so it is expected that the δ-V curves are the same. The estimated δ-V curve is given in figure 5 of KD 213. This δ-V curve is copied as figure 3. The rotor starts turning out of the wind at a wind speed of about 6 m/s. The rotor is turned out of the wind 30° at a wind speed of 9.5 m/s. It is assumed that the ideal curve is followed for wind speeds above 9.5 m/s.

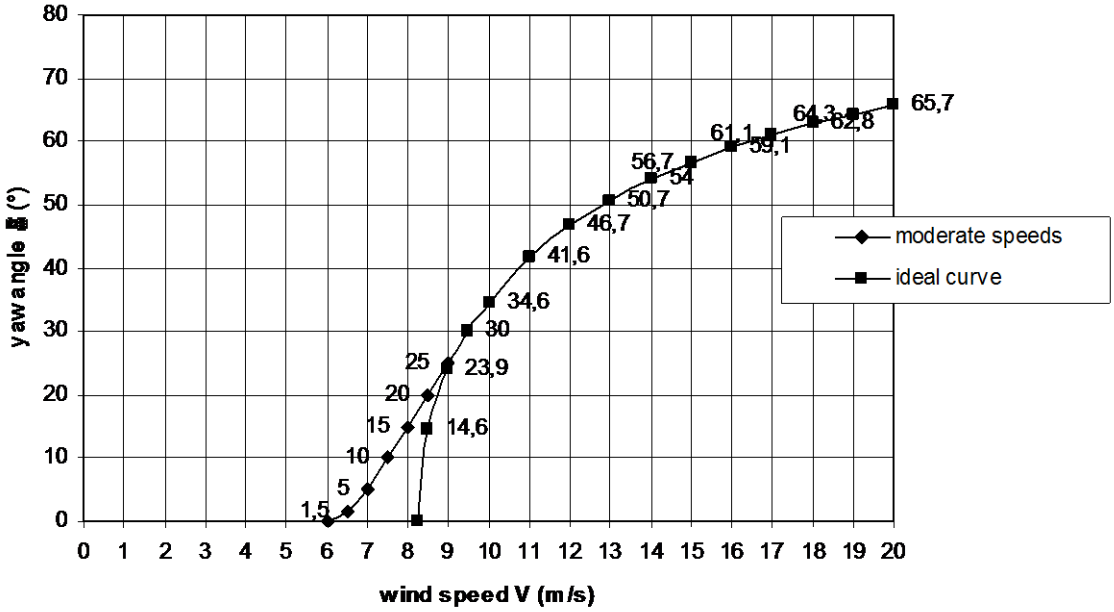

fig. 3 Estimated δ-V curve for the VIRYA-3.1 windmill

The P-n curves are normally used to check the matching with the P_{mech}-n curve of the generator and to determine the P_{el}-V curve. However, measured characteristics of the 34-pole PM-generator are not available so this procedure can’t be performed. But it is still useful to make the P-n curves because lines of a constant frequency can be drawn in this graph.

Because the P-n curve for low values of λ appears to lie very close to each other, the P-n curves are not determined for very low values of λ. The P-n curves are determined for C_p values belonging to λ is 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5 and 10.4 (see figure 1). The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9 and 9.5 m/s.

Substitution of R = 1.55 m in formula 7.1 of KD 35 gives:

$$n = 6.161 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of ρ = 1.2 kg / m^3 en R = 1.55 m in formula 7.10 of KD 35 gives:

$$P = 4.529 * C_p * \cos^3\delta * V^3 \qquad \text{(W)} \qquad (9)$$

For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For wind speeds higher than 6 m/s, the yaw angle δ is taken into account.

		V = 3 m/s $\delta = 0°$		V = 4 m/s $\delta = 0°$		V = 5 m/s $\delta = 0°$		V = 6 m/s $\delta = 0°$		V = 7 m/s $\delta = 5°$		V = 8 m/s $\delta = 15°$		V = 9 m/s $\delta = 25°$		V = 9.5 m/s $\delta = 30°$	
λ	Cp	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)
3.5	0.15	64.7	18.3	86.3	43.5	107.8	84.9	129.4	146.7	150.4	230.4	166.6	313.5	175.9	368.7	177.4	378.3
4.5	0.29	83.2	35.5	110.9	84.1	138.6	164.2	166.3	283.7	193.3	445.4	214.2	606.0	226.1	712.8	228.1	731.4
5.5	0.39	101.7	47.7	135.5	113.0	169.4	220.8	203.3	381.5	236.3	599.0	261.8	815.0	276.4	958.6	278.8	983.6
6.5	0.42	120.1	51.4	160.2	121.7	200.2	237.8	240.3	410.9	279.3	645.0	309.5	877.7	326.7	1032.3	329.5	1059.3
7.5	0.39	138.6	47.7	184.8	113.0	231.0	220.8	277.2	381.5	322.2	599.0	357.1	815.0	376.9	958.6	380.2	983.6
8.5	0.305	157.1	37.3	209.5	88.4	261.8	172.7	314.2	298.4	365.2	468.4	404.7	637.4	427.2	749.6	430.8	769.2
9.5	0.165	175.6	20.2	234.1	47.8	292.6	93.4	351.2	161.4	408.1	253.4	452.3	344.8	477.4	405.5	481.5	416.1
10.4	0	192.2	0	256.3	0	320.4	0	384.4	0	446.8	0	495.1	0	522.6	0	527.2	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-3.1 rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which is going through the tops of the P_{mech}-n curves is also given in figure 4.

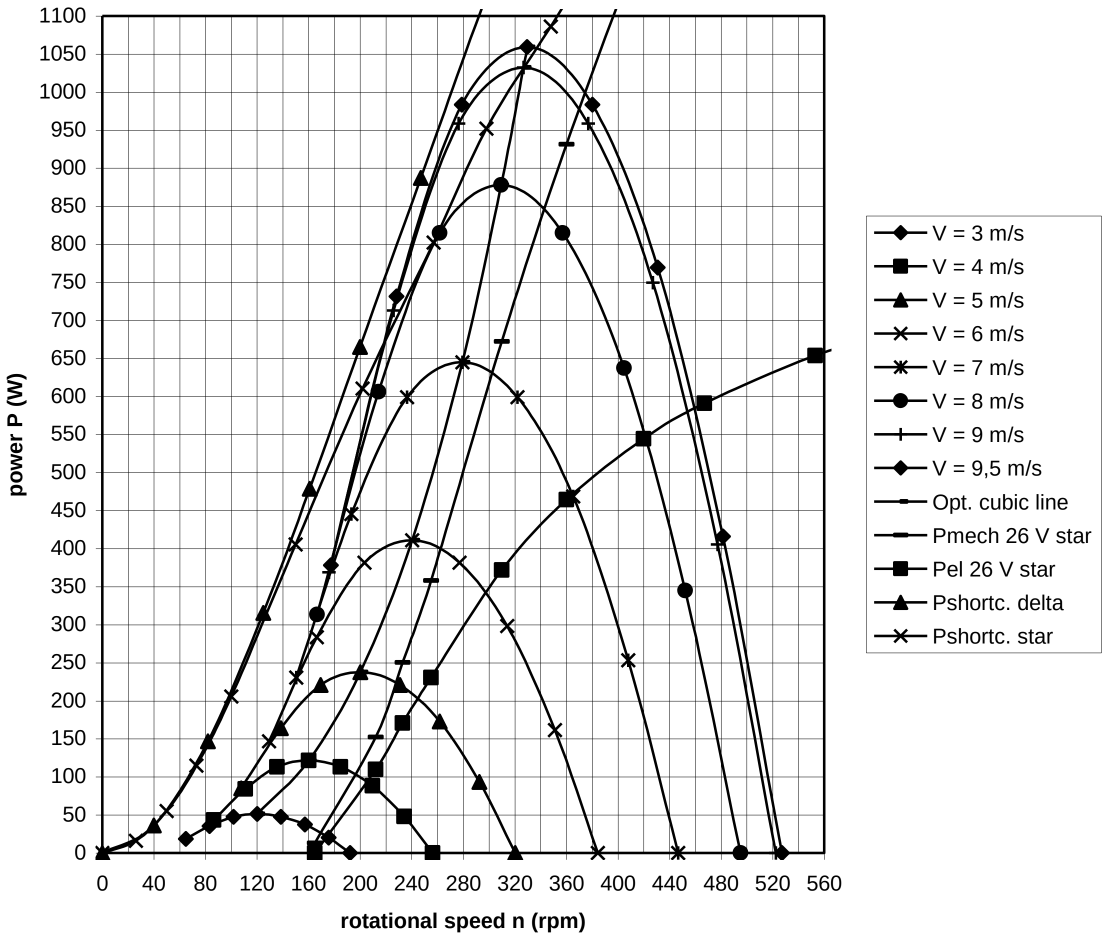

fig. 4 P-n curves, optimum cubic line, P_{mech}-n and P_{el}-n curves for 26 V star for a modified 115/200 V winding and P-n curves for short-circuit in delta and star

For charging of a 24 V battery, the original 230/400 V winding has to be modified into a 115/200 V winding. How this is done is explained in report KD 341 (ref. 5). The winding has to be rectified in star for 24 V battery charging. Rectification of 3-phase generators is explained in report KD 340 (ref. 6).

The generator has been measured for the original 230/400 V winding for 52 V star. All the characteristics for the original winding for 52 V star will be the same as the characteristics for the modified 115/200 V winding for 26 V star. The P_{mech}-n and P_{el}-n curves of the generator for 26 V star, for the modified 115/200 V winding, are also plotted in figure 4. A voltage of 26 V is the average charging voltage for a 24 V battery.

The generator has been measured for short-circuit in delta and for short-circuit in star. Short-circuited in delta is advised because the maximum torque level for short-circuit in delta is higher than for short-circuit in star. Short-circuit in delta is identical to short-circuit in star if the star point is short-circuited too. The P-n curves for short-circuit in delta and for short-circuit in star are also plotted in figure 4.

The point of intersection of the P_{mech}-n curve of the generator with the P-n curve of the rotor for a certain wind speed, gives the working point for that wind speed. The electrical power P_{el} for that wind speed is found by going down vertically from the working point up to the point of intersection with the P_{el}-n curve. The values of P_{el} found this way for all wind speeds, are plotted in the P_{el}-V curve (see figure 5). The charging voltage at high powers will be somewhat higher than the average charging voltage of 26 V and therefore the generator efficiency will be somewhat higher too. This results in a somewhat higher electrical power. The P_{el}-V curve is corrected for this effect for high wind speeds.

The matching of rotor and generator is good because the P_{mech}-n curve of the generator is lying close to the optimum cubic line for wind speeds in between 4 and 9.5 m/s. In the P_{el}-V curve it can be seen that the maximum power is 500 W and that supply of power starts already at a wind speed of 2.7 m/s ($V_{cut\ in}$ = 2.7 m/s). This is rather low and therefore the windmill can be used in regions with low wind speeds. In chapter 4 it was calculated that V_{start} = 2.7 m/s so there is no hysteresis in the P_{el}-V curve.

The P-n curve for short-circuit in delta is lying left from the P-n curve of the rotor for V = 9.5 m/s and higher. This means that the rotor will slow down to almost stand still for every wind speed if short-circuit in delta is made. The P-n curve for short-circuit in star is touching the P-n curve of the rotor for V = 8 m/s so the rotor will only stop for wind speeds below 8 m/s. So making short-circuit in star to stop the rotor is not allowed because the rotor will not stop at high wind speeds and after a certain time the winding will burn. Short-circuit in star is the same as short-circuit in delta if the star point is short-circuited too.

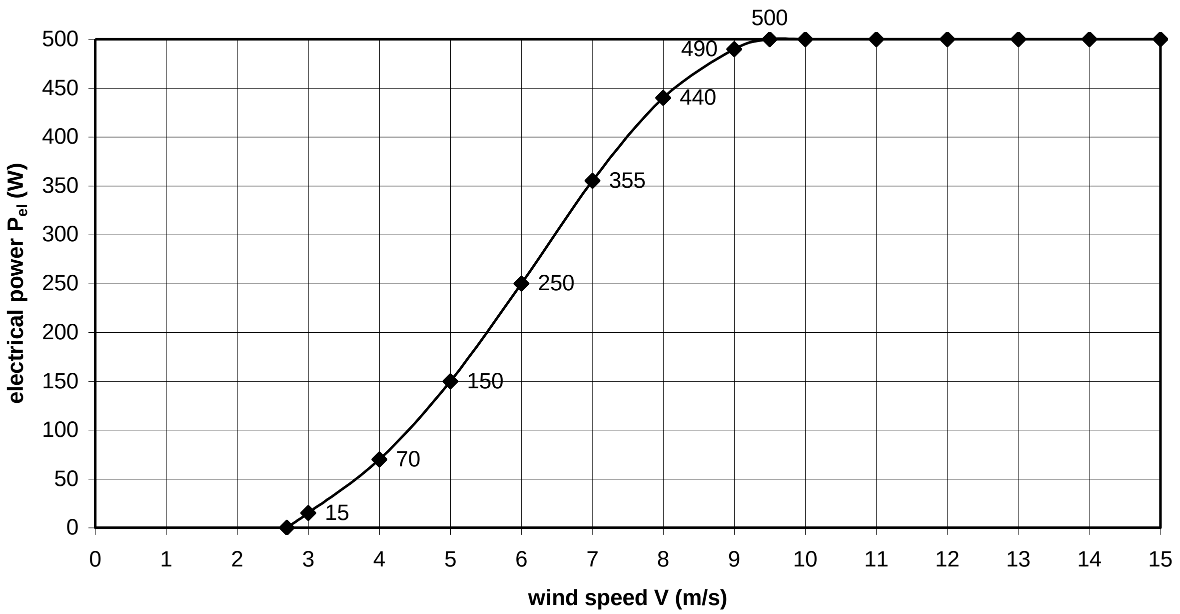

fig. 5 P_{el}-V curve of the VIRYA-3.1 windmill for 24 V battery charging for a modified 115/200 V winding and rectification in star

6 Determination of the P-n curves and the P_{el}-V curve for 13 V delta

If there is a need for 12 V battery charging in stead of 24 V battery charging, it might be possible to use the modified 115/200 V winding with rectification in delta. The characteristics for 26 V delta for the original 230/400 V winding are given in report KD 78 (ref. 1). The characteristics for the modified 115/200 V winding will be the same for 13 V delta. Figure 4 is now copied as figure 6 but the curves for 26 V star are replaced by the curves for 13 V delta (13 V is the average charging voltage for a 12 V battery).

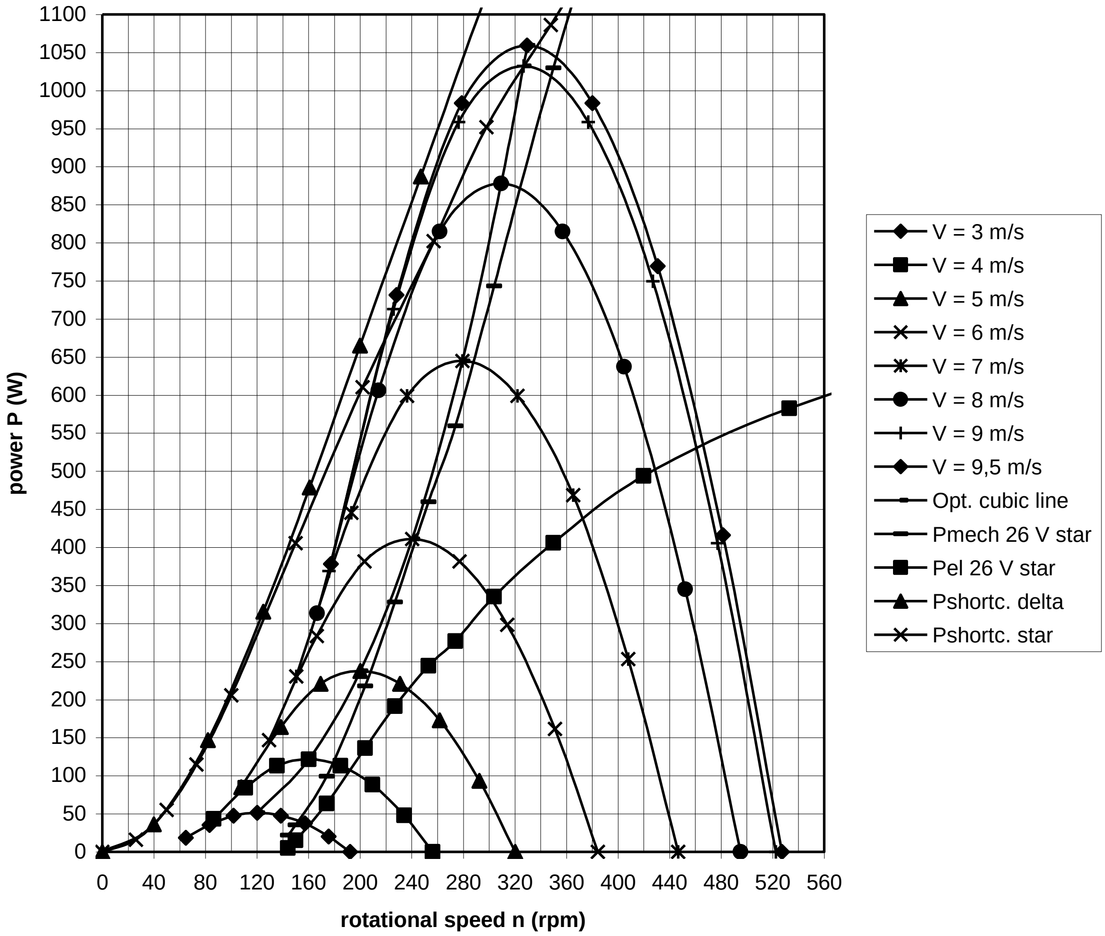

fig. 6 P-n curves, optimum cubic line, P_{mech}-n and P_{el}-n curves for 13 V delta for a modified 115/200 V winding and P-n curves for short-circuit in delta and star

In figure 6 it can be seen that the matching is even better as for 26 V star. An advantage of delta rectification is that making short circuit in delta requires no extra line to the short-circuit switch and that a 2-pole short-circuit switch can be used.

The P_{el}-V curve for 13 V delta is derived from figure 6 using the method given in chapter 5. The P_{el}-V curve is given in figure 7.

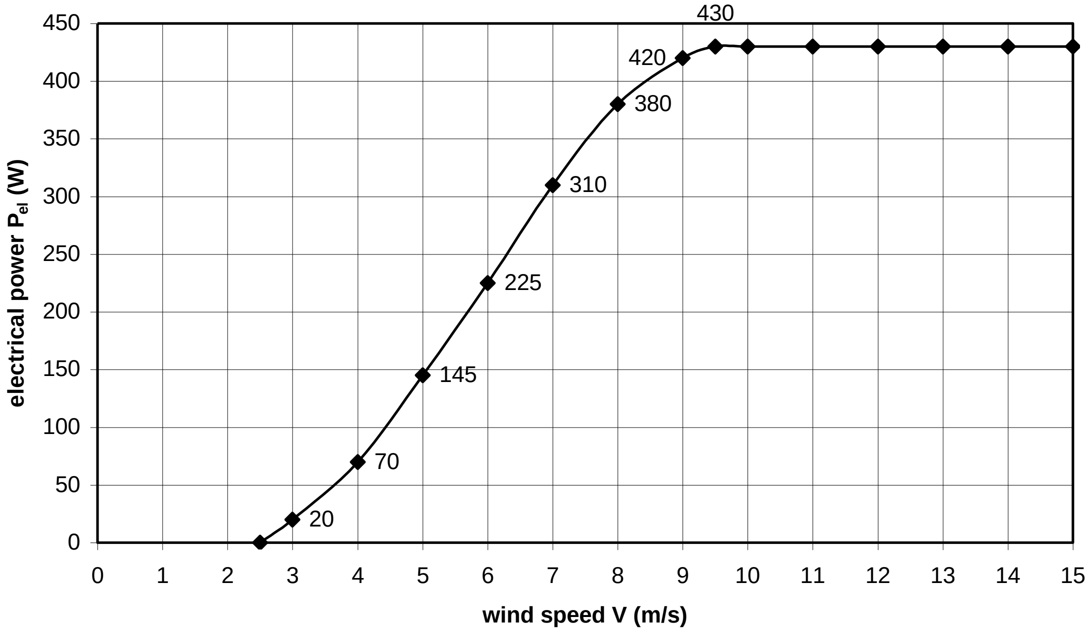

fig. 7 P_{el}-V curve of the VIRYA-3.1 windmill with V_{rated} = 9.5 m/s for 12 V battery charging and rectification of the modified 115/200 V winding in delta

In figure 7 it can be seen that the maximum power is 430 W which is 70 W lower than for 26 V star. The whole P_{el}-V curve is lying lower than for 26 V star except for wind speeds below 4 m/s. This is because of the lower generator efficiency in delta. The cut-in wind speed lies at about 2.5 m/s which is a little lower than for 26 V star.

It is possible to use the original 230/400 V winding for 24 V battery charging if the winding is rectified in delta. For this situation the same P_{el}-V curve will be found as given in figure 7. So the power will be lower as for the modified 115/200 V winding, rectified in star. Another disadvantage is that the starting behaviour in delta is worse than in star because the unloaded P_{mech}-n curve of the generator is rising faster in delta than in star. This is because in delta, higher harmonic currents can circulate in the winding.

It is also possible to use the original 230/400 V winding for 48 V battery charging if the winding is rectified in star. An advantage of the high voltage is that the heat losses in the lines in between generator and battery are a factor four lower if the same wire thickness is used. Disadvantages are that now a more complicated 48 V battery charge controller is needed and that almost no 48 V equipment is available.

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Appendix 1

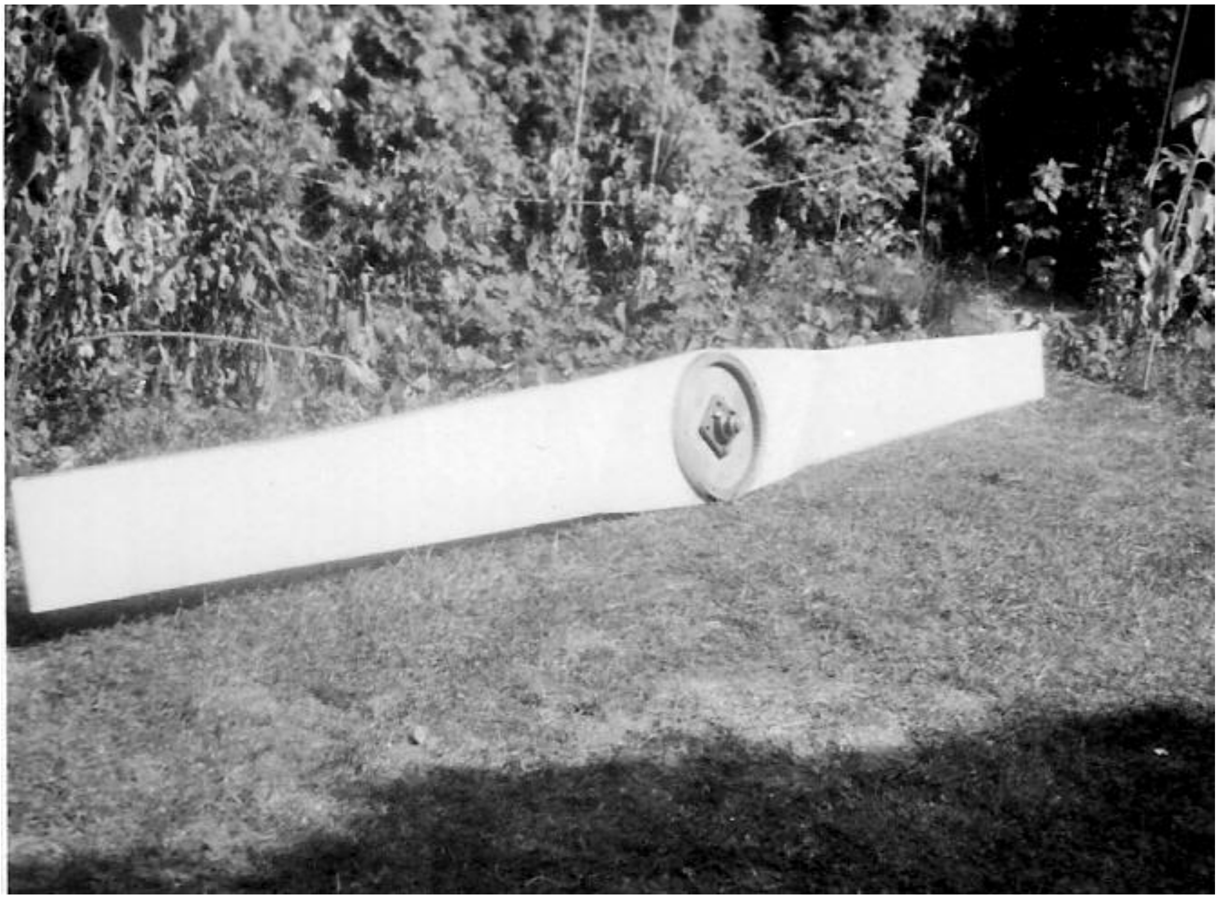
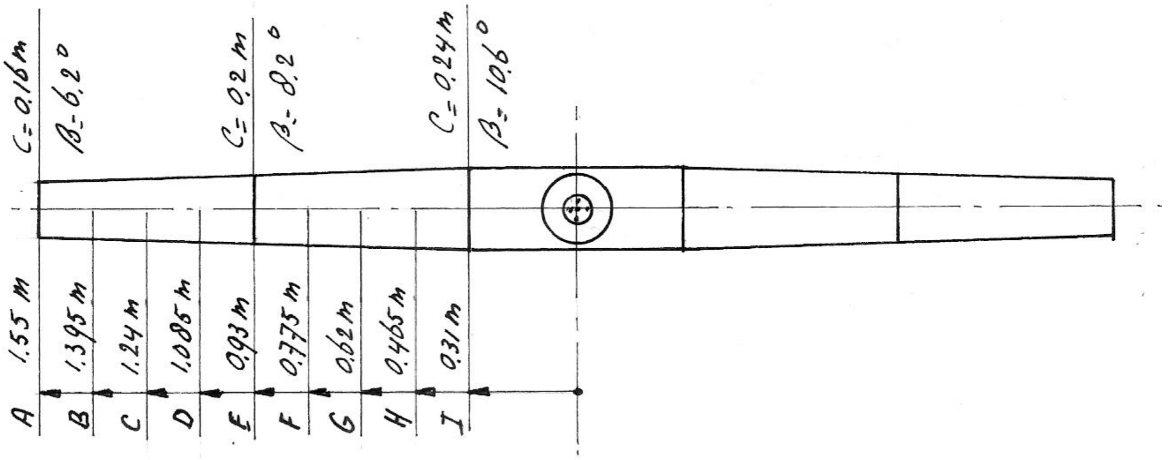


Photo Roofmate rotor with D = 3.2 m and λ_d = 8 made in 1977

Appendix 2



Sketch of the VIRYA-3.1 rotor